Event Logic and Abstract Programs

(Workshop on Event-based Semantics, St. Louis)

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1 Introduction

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Extracts will send messages, and have state.

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Caveat: The "axioms" are not all consistent with each other. E.g. we can't use both
@i **effect of** k **is** x:=x+1
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We use these higher level abstractions to reason more abstractly about consensus, fault-tolerance, security, etc. Formal methods can knit a complex argument together.

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What are useful abstract programs?

2 Outline

1. Message automata

2. Event structures

3. Authentication Example

4. Event classes

5. Abstract programs

3 Message Automata

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The join is the basic composition operator on automata.

Semantics: set of *event structures* that are *consistent with* A.

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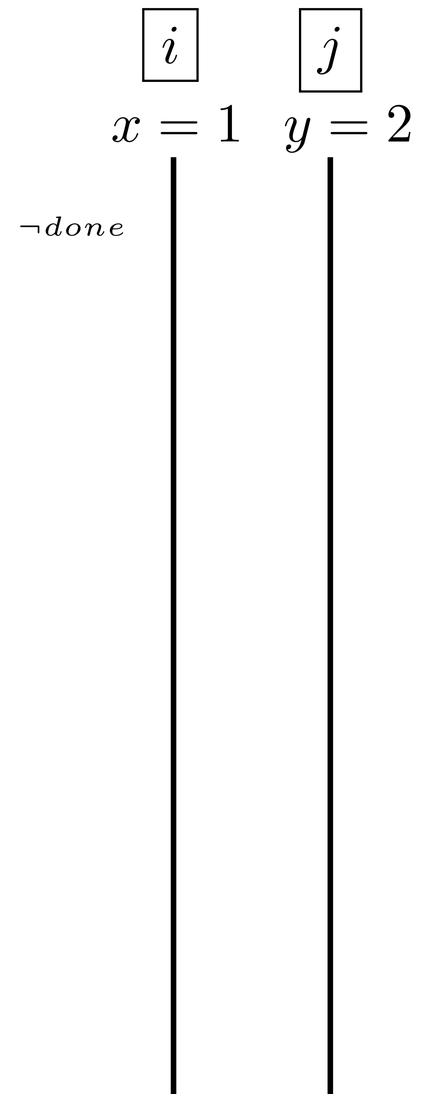

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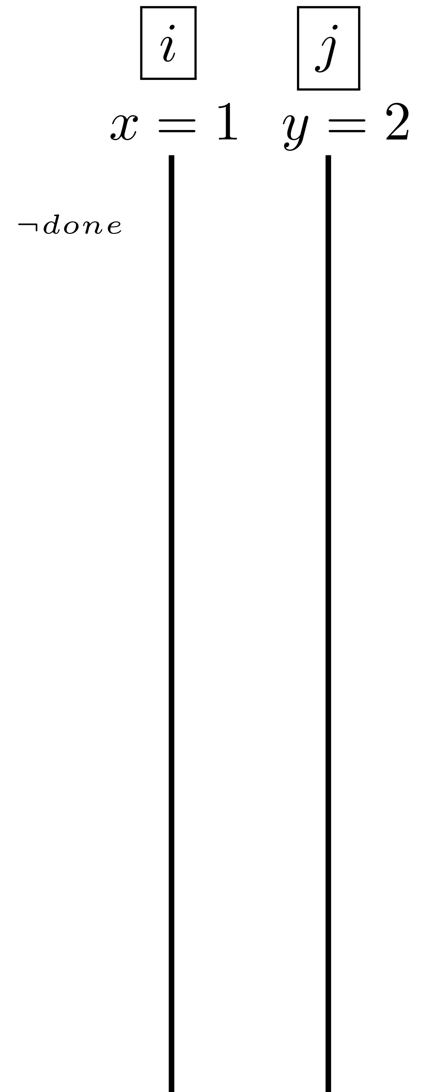

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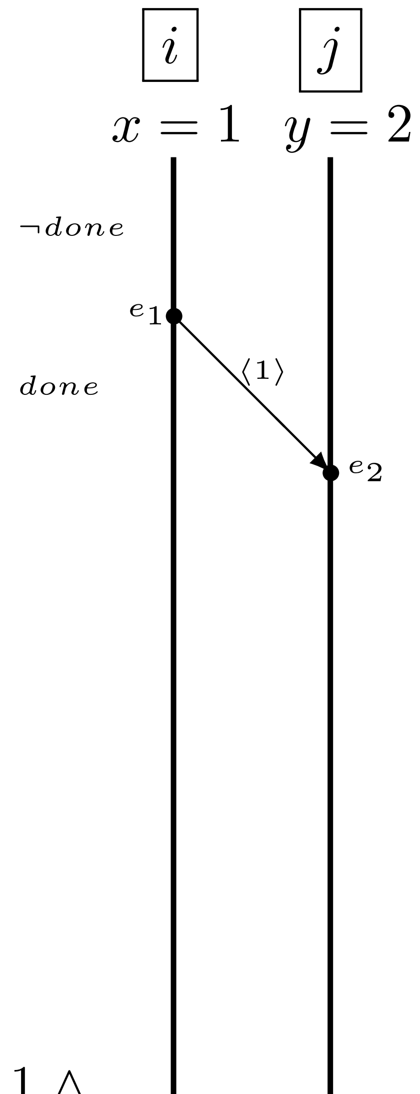

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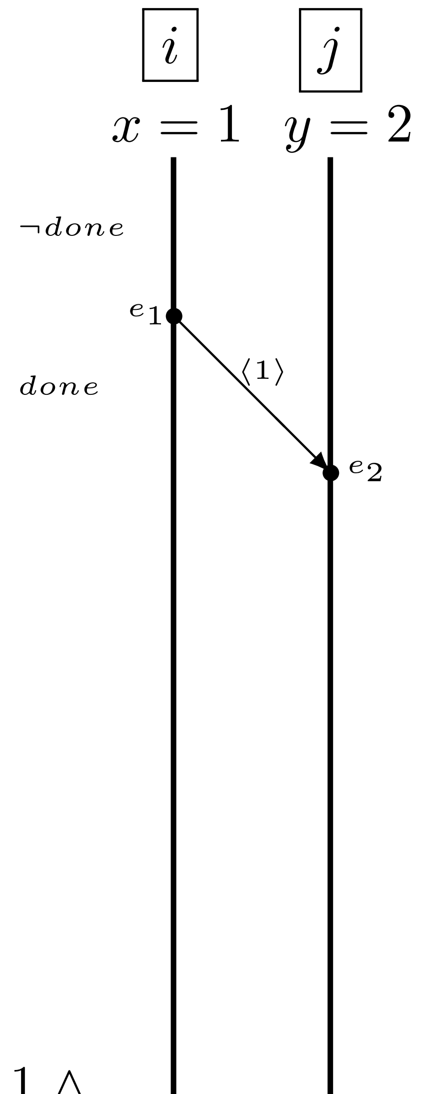

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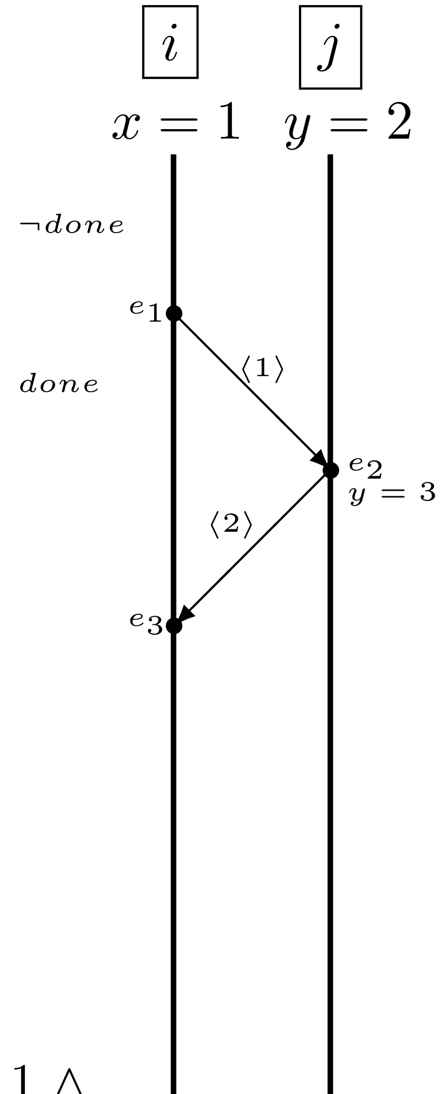

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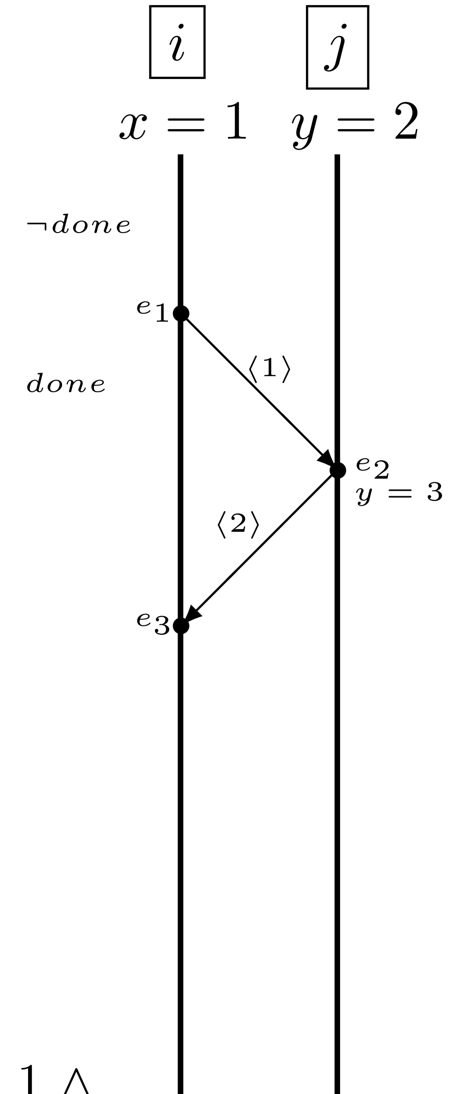

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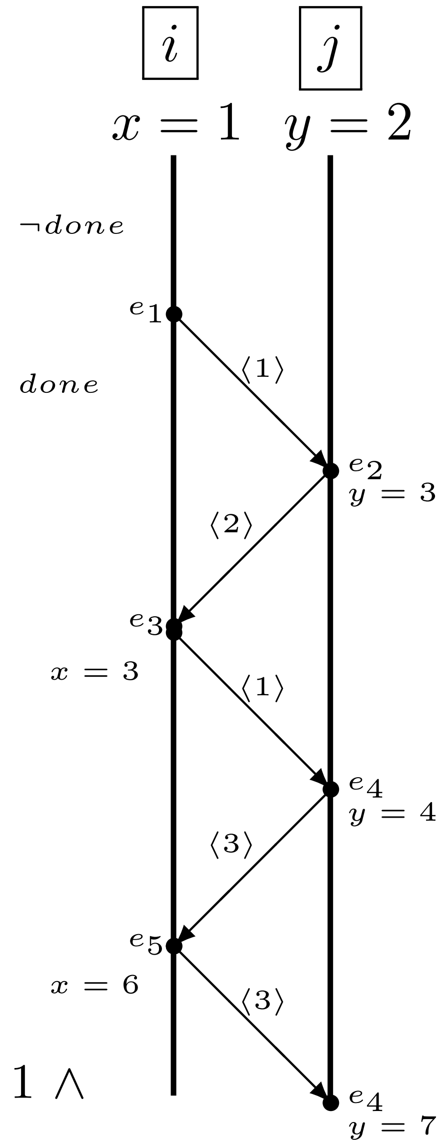

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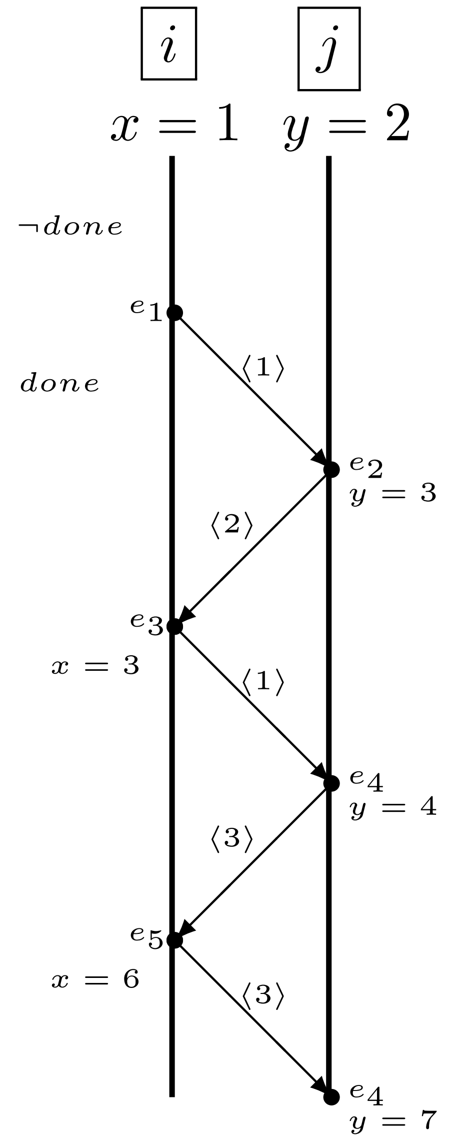
$\forall e, e'@i.\ e < e' \Rightarrow x\ \textbf{when}\ e \leq x\ \textbf{when}\ e'$



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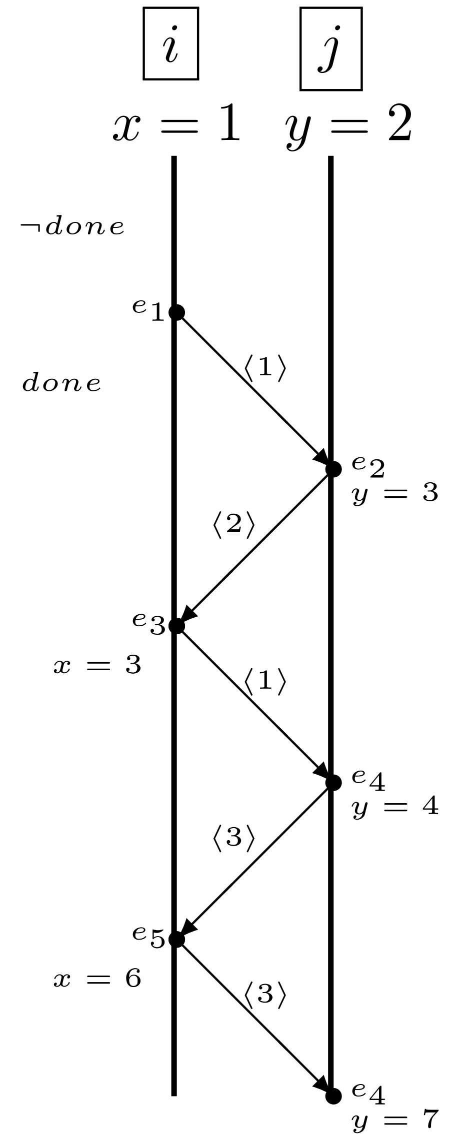
$\forall e, e'@i.\ e < e' \Rightarrow x \textbf{ when } e \leq x \textbf{ when } e'$

Notation:

$\forall e@i.\ P[e] \equiv \forall e.\ ((loc(e) = i) \Rightarrow P[e])$



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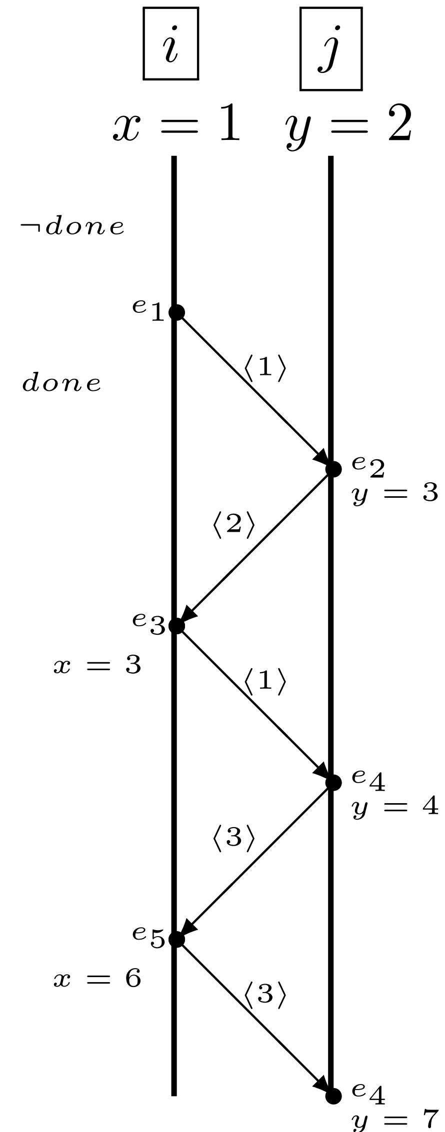

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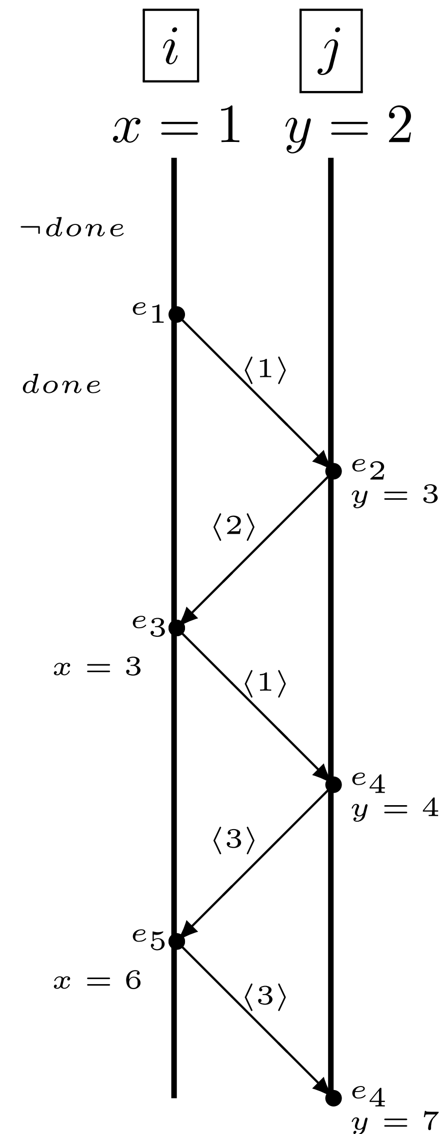
But we need more constraints ...



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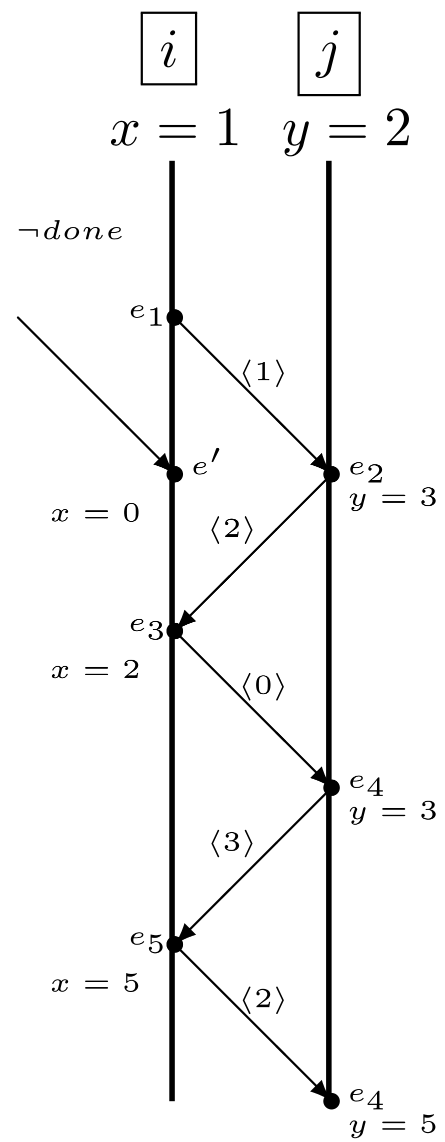

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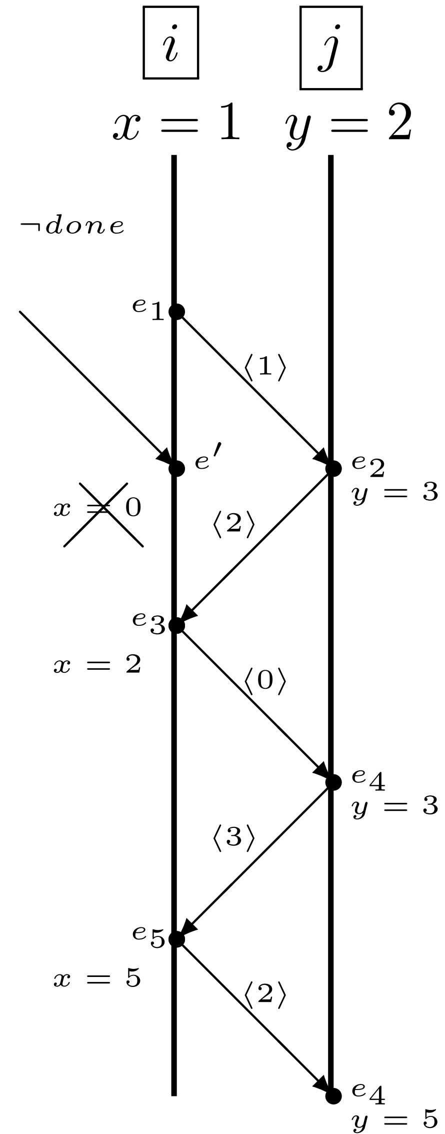

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Each clause C constrains the *event structures* that are *consistent* with C.

This frame clause:

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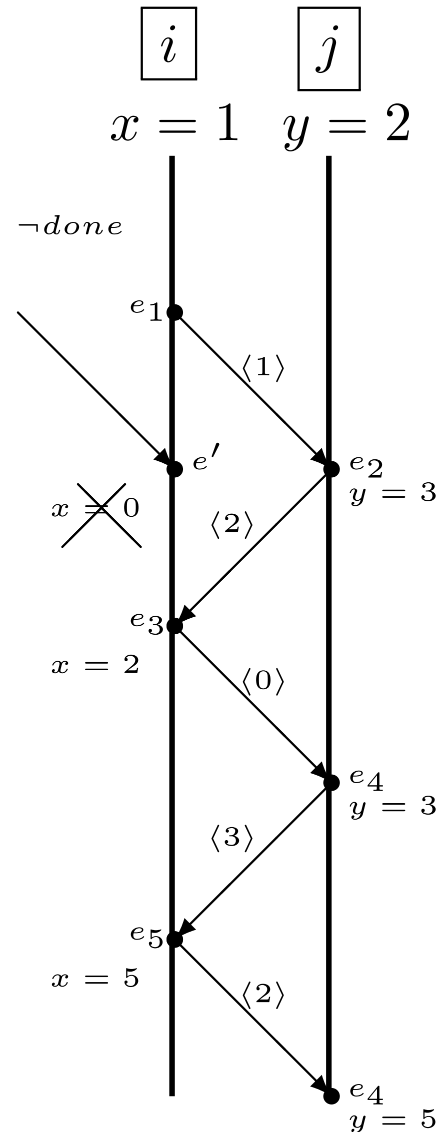

Add constraints in M_2 (forming automaton $M_1 \oplus M_2$)

message automaton M_2

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only [rcv(l_1, num)] **affect** y **at location** j

only [internal(i, go), rcv(l_2,num)] **send on link** l_1 **with tag** num

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M_1 and M_2 are *compatible* (obey each other's constraints), so $M_1 \oplus M_2$ is *feasible.*

4 Event Structures

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1. On any link, a single event sends boundedly many messages.
2. Events at a single location are totally ordered.
3. Causal order is (strongly) well-founded.
4. The location of the sender of an event is the source of the link on which the message was received; formally:
   $\forall e : E.\ isrcv(e) \Rightarrow loc(\mathbf{sender}(e)) = src(link(e))$
5. Links deliver messages in FIFO (first in first out) order
   $\forall e_1, e_2 : E.\ link(e_1) = link(e_2) \Rightarrow \mathbf{sender}(e_1) < \mathbf{sender}(e_2) \Rightarrow e_1 < e_2$
6. State variables change only at events, so that:
   $\forall e : E. \neg first(e) \Rightarrow (x\ \mathbf{when}\ e) = (x\ \mathbf{after}\ pred(e))$

Realizability

Clause constraints from $M_1 \oplus M_2$ suffice to prove (by induction on $<$) properties like the invariant

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Theorem

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Definition

M **realizes** $\psi \equiv$

$Feasible(M) \ \wedge\ \forall es.\ Consistent(es, M) \ \Rightarrow\ es \models \psi$

5 Example: Authentication Protocol

Initiator(A)	Network	Responder(B)
New(n)		
Send	$\xrightarrow{A, B, n}$	Receive(n) from A
		$sig = \text{Sign}_B(n)$
Receive(sig) from B	$\xleftarrow{B, A, sig}$	Send
Verify(B, sig, n)		

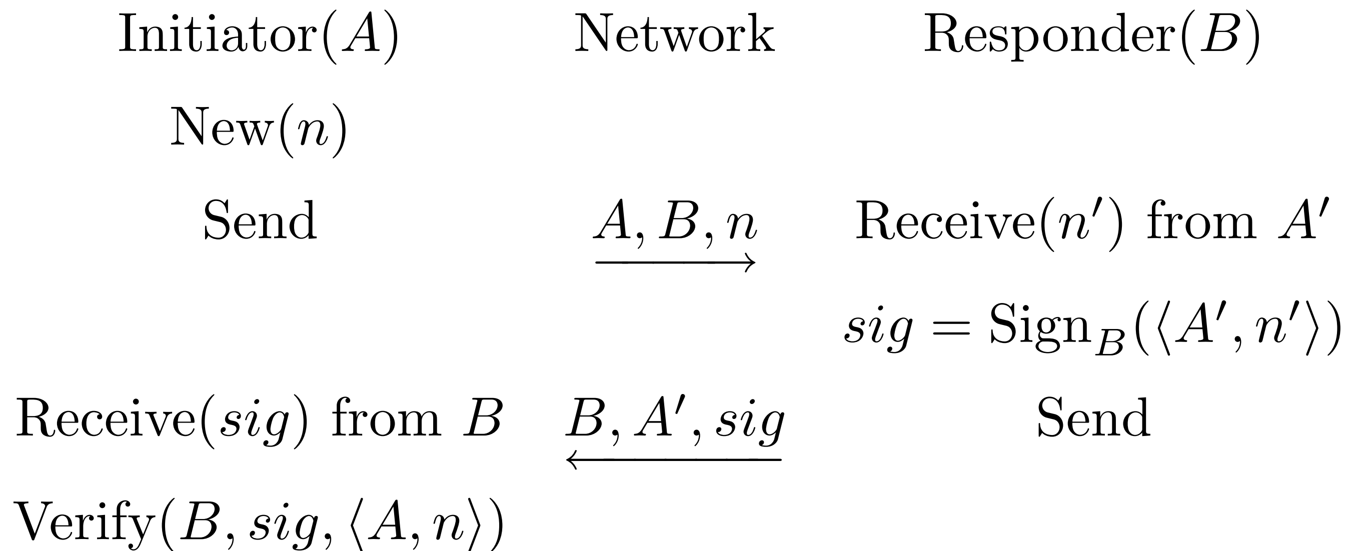
Spec: *Matching conversation*

6 Protocol Bug

Initiator(A)	Network	Adversary	Responder(B)
New(n)			
Send	$\xrightarrow{A, B, n}$	$\xrightarrow{A', B, n}$	Receive(n) from A'
			$sig = \text{Sign}_B(n)$
Receive(sig) from B	$\xleftarrow{B, A, sig}$	$\xleftarrow{B, A', sig}$	Send
Verify(B, sig, n)			

sig is correct signature for B of n, but the *Send* and *Receive* events do not form a matching conversation.

7 Correct Authentication Protocol



Unless $A' = A$ and $n' = n$, sig will not be verified, so the *Verify* event does imply a matching conversation.

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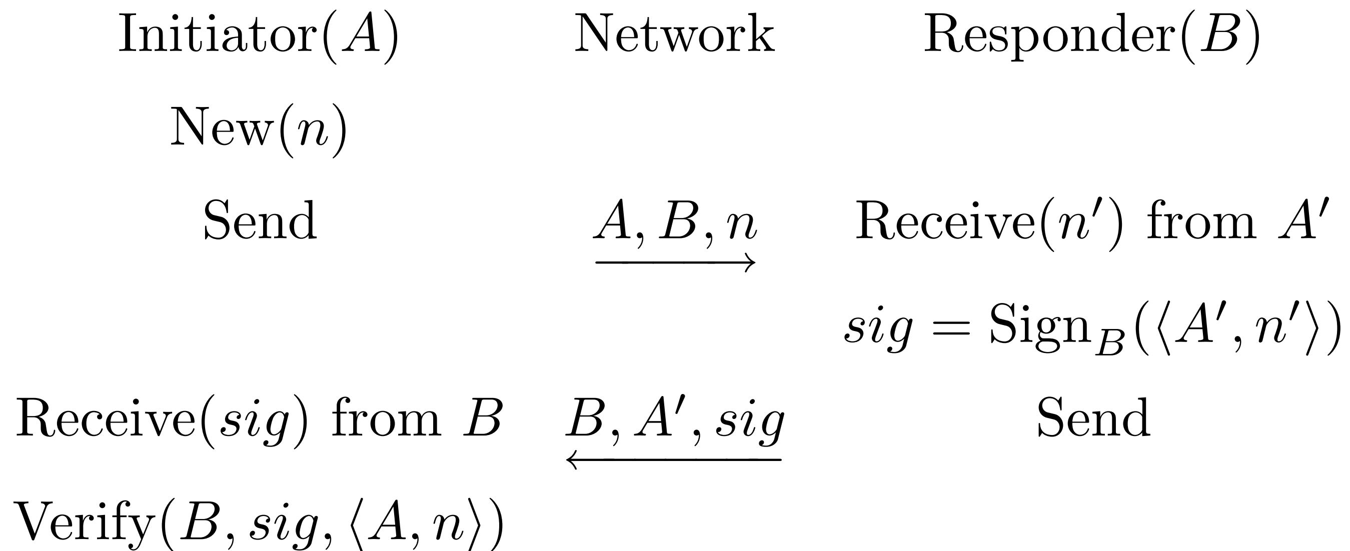

Initiator(A)	Network	Responder(B)
New(n)		
Send	$\xrightarrow{A, B, n}$	Receive(n') from A'
		$sig = \text{Sign}_B(\langle A', n' \rangle)$
Receive(sig) from B	$\xleftarrow{B, A', sig}$	Send
Verify($B, sig, \langle A, n \rangle$)		

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Can we reason formally at an abstract level?

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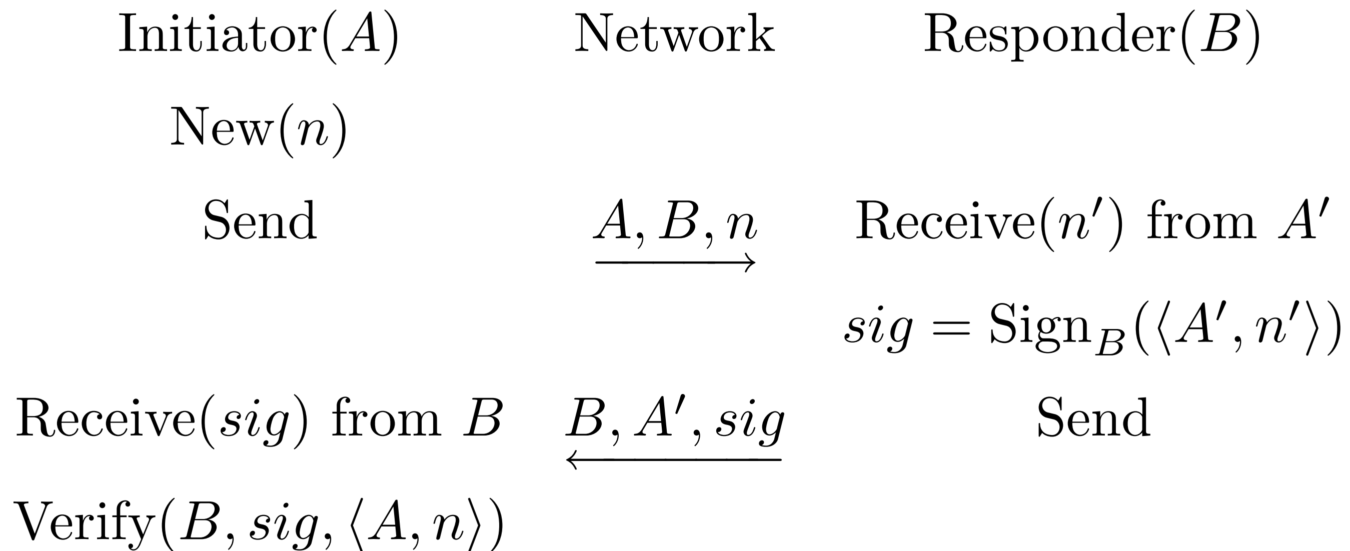

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Can we reason formally at an abstract level?

Yes, by using *Event classes* and *sequential laws*

8 Event Classes

An *event class* X (of type T) is a partial function from events to values of type T.

$$X : E \rightarrow T + Unit$$

Notation:

- domain $E(X)$
- value $X(e)$
- $e \in X(v)$ means $e \in E(X)$ and $X(e) = v$.
- $X(v) \Rightarrow Y(f(v))$ means

$$\forall e.\ e \in X(v) \Rightarrow e \in Y(f(v))$$

This says that any event in class X is also in class Y and its value in class Y is a function of its value in class X

9 Sequential Law

The *sequential law* $A; B$ is the conjunction of the two constraints

$$A \hookrightarrow B \qquad \text{defined as}$$

$$\forall e, a.\ e \in A(a) \Rightarrow \exists e', b.\ e <_c e' \ \wedge \ e' \in B(a, b)$$

and

$$A \hookleftarrow B \qquad \text{defined as}$$

$$\forall e', a, b.\ e' \in B(a, b) \Rightarrow \exists e.\ e <_c e' \ \wedge \ e \in A(a)$$

The *iterated sequential law* $A; B; C$ is the conjunction of the laws $A; B$ and $B; C$

$$A \hookrightarrow \quad B \quad \hookrightarrow C$$

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10 Initiator as Abstract Program

The initiator role = a sequence of four event classes of the form:

$$I_1(A,k,n); I_2(A,k,n,B); I_3(A,k,n,B,sig); I_4(A,k,n,B,sig)$$

where (each instance thread of the role has a number k) and

$$I_1(A,k,n) \Rightarrow \mathit{Nonce}(n) \tag{1}$$

$$I_2(A,k,n,B) \Rightarrow \mathit{Send}(A,B,n) \tag{2}$$

$$I_3(A,k,n,B,sig) \Rightarrow \mathit{Receive}(B,A,sig) \tag{3}$$

$$I_4(A,k,n,B,sig) \Rightarrow \mathit{Verify}(B,\langle A,n\rangle,sig) \tag{4}$$

and

$$I_1; I_2$$

$$I_2 \hookleftarrow I_3 \hookleftarrow I_4$$

11 Responder as Abstract Program

The responder role has the form:

$$R_1(B, k, A, n); R_2(B, k, A, n, sig); R_3(B, k, A, n, sig)$$

where

$$R_1(B, k, A, n) \Rightarrow \mathit{Receive}(A, B, n) \tag{5}$$

$$R_2(B, k, A, n, sig) \Rightarrow \mathit{Sign}(B, \langle A, n \rangle, sig) \tag{6}$$

$$R_3(B, k, A, n, sig) \Rightarrow \mathit{Send}(B, A, sig). \tag{7}$$

satisfies the sequential law

$$R_1; R_2; R_3$$

and

$$e \in \mathit{Sign}(B, \langle A, n \rangle, sig) \Rightarrow \exists k.\ e \in R_2(B, k, A, n, sig)$$

12 Nonce and Signature

The nonce class *Nonce* satisfies:

$$e \in Nonce(n) \ \wedge \ e' \in X(\ldots, n, \ldots) \Rightarrow e \leq_c e'$$

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Signing event $e \in \mathit{Sign}(B, v, sig)$ (signer, data, signature) is also a nonce if it is the first time B signs v.

Verify event $e \in \mathit{Verify}(B, v, sig)$ represents the successful verification.

Sign and *Verify* are related by the law:

$$e \in \mathit{Sign}(signer, v, sig) \hookleftarrow e' \in \mathit{Verify}(signer, v, sig)$$

which says that a successful verify event must be preceded by a matching signing event.

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$$
\begin{aligned}
e_1 \in I_1(A, k, n) \quad &\wedge \quad e_1 \in \mathit{Nonce}(n) \\
e_2 \in I_2(A, k, n, B) \quad &\wedge \quad e_2 \in \mathit{Send}(A, B, n) \\
e_3 \in I_3(A, k, n, B, sig) \quad &\wedge \quad e_3 \in \mathit{Receive}(B, A, sig) \\
e_4 \in I_4(A, k, n, B, sig) \quad &\wedge \quad e_4 \in \mathit{Verify}(B, \langle A, n \rangle, sig)
\end{aligned}
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e_4 implies that there exists event $e_2' <_c e_4$ with

$$e_2' \in \mathrm{Sign}(B, \langle A, n \rangle, sig) \ \wedge \ e_2' \in R_2(B, k', A, n, sig)$$

for some k'

From the sequential law $R_1; R_2; R_3$ events e'_1 and e'_3 exist with $e'_1 <_c e'_2 <_c e'_3$ and

$$\begin{aligned} e'_1 \in R_1(B, k', A, n) &\quad \wedge \quad e'_1 \in \mathit{Receive}(A, B, n) \\ e'_2 \in R_2(B, k', A, n, sig) & \\ e'_3 \in R_3(B, k', A, n, sig) &\quad \wedge \quad e'_3 \in \mathit{Send}(B, A, sig) \end{aligned}$$

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Already have $e_2 \in \mathit{Send}(A, B, n) \;\wedge\; e_3 \in \mathit{Receive}(B, A, sig)$

Events e_2, e'_1, e'_3, e_3 form a matching conversation between A and B, provided that we can show

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Events e_2, e'_1, e'_3, e_3 form a matching conversation between A and B, provided that we can show

$$e_2 <_c e'_1 <_c e'_3 <_c e_3$$

$e_1 <_c e'_1$ follows from Nonce property, but this also implies that $e_2 <_c e'_1$ because initiator A does not send nonce n before e_2.

We already have $e'_1 <_c e'_3$ from the sequential law for responder B.

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This follows from the fact that $e'_2 \in \mathrm{Sign}(B, \langle A, n \rangle, sig)$ is the only signing event at location B that includes nonce n, and the signature sig is therefore also a nonce, which is not sent by responder B until event e'_3. Since $e_3 \in \mathit{Receive}(B, A, sig)$ we must have $e'_3 <_c e_3$. QED

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Can we derive code from this kind of abstract argument?

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(Work in progress) Behaviors we must realize:

- Recognize events (and their values)
- Generate new events (by sending messages, choosing nonces, etc.)

14 Local (Programmable) Classes

A *local* class X depends only on locally available information.
Events $E(X)$ may include events at several locations, but whether $e \in E(X)$ and the value $X(e)$ are functions of $loc(e)$,$kind(e)$,$val(e)$, and finitely many state variables x **when** e.

Programmable classes are

- basic X: $E(X)$ = finite set of location $\times$ kinds, $X(e) = val(e)$.
- $X + Y$ where X and Y are programmable.
- $f \circ X^*$, $e \in E(X)$, value $f([X(e') \| e' \leq e])$

Theorem: Programmable $\Rightarrow$ Local
So events in a programmable class can be recognized and assigned values by a program (that we can synthesize).

15 Propagation Rules

A protocol or distributed algorithm does more than recognize events.

In order to satisfy constraints like

$$A \hookrightarrow B$$

it must *propagate* information from events of class A to events of class B. If B is a class of receive events, this can be accomplished by a message automaton that recognizes A events and sends messages containing the values of these events.

Basic propagation rule:

$$A \Rightarrow \mathrm{rcv}(dst, tg) \qquad \text{says:}$$

$$\forall e \in A(a)\ \exists e'.\ \mathbf{sender}(e') = e\ \wedge\ val(e') = a\ \wedge\ loc(e') = dst \ldots$$

16 Abstract Programs

Abstract program = set of propagation rules
where all classes are programmable.

From basic programmable class combinators we can derive more, e.g. $A;_R B$ where $e \in A;_R B(\langle a, b \rangle)$ iff

$$e \in B(b) \ \wedge \ R(a,b) \ \wedge \ \exists e' \leq e.\ e' \in A(a)\ s.t.\ e' \text{ is most recent}$$

If C is a class of "configuration" input events, then a function of C^* defines the "current configuration" *Config*. Then to define how an input event in class B should respond in the current configuration, use a propagation rule of the form

$$\mathit{Config}; B \Rightarrow \ldots$$

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Result will be a high-level programming language that is easy to reason about and compiles into efficient code.

18 Virtual Nonce Serve

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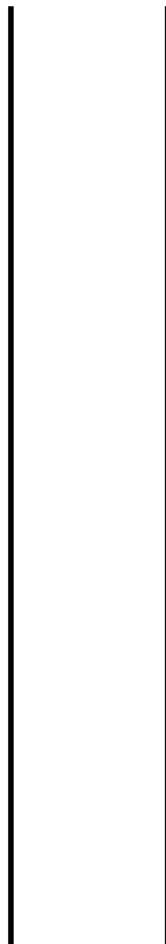

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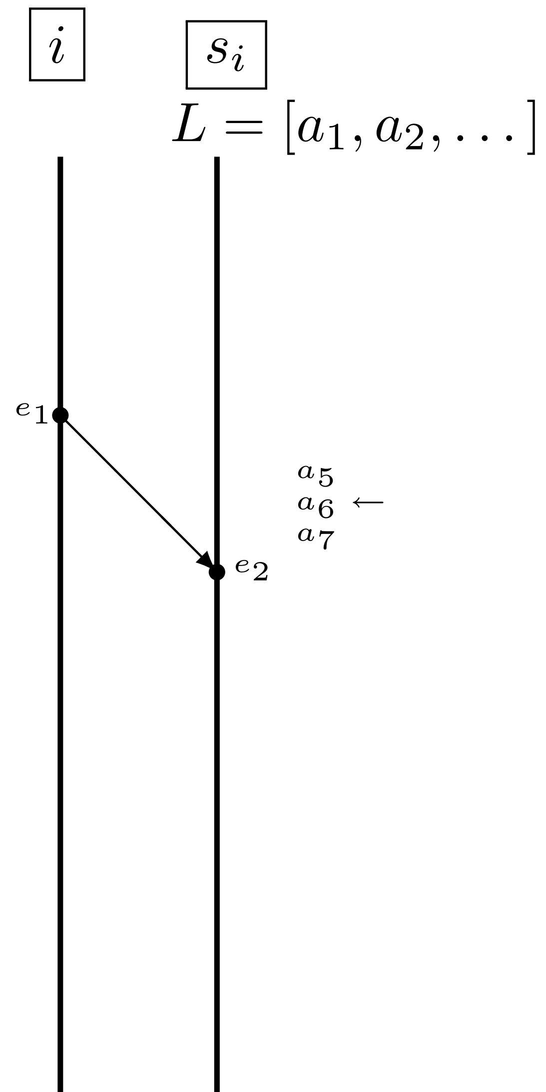

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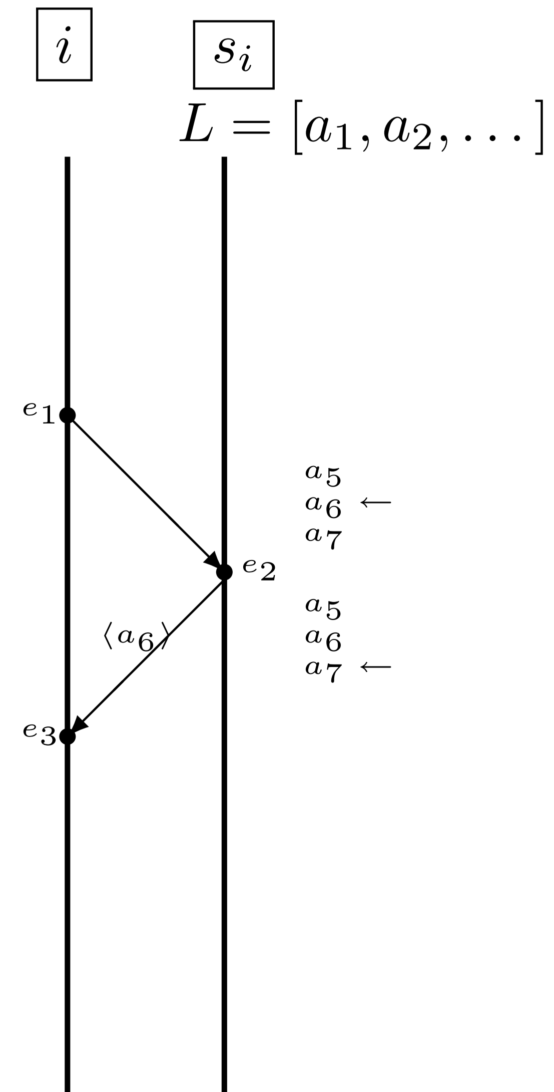

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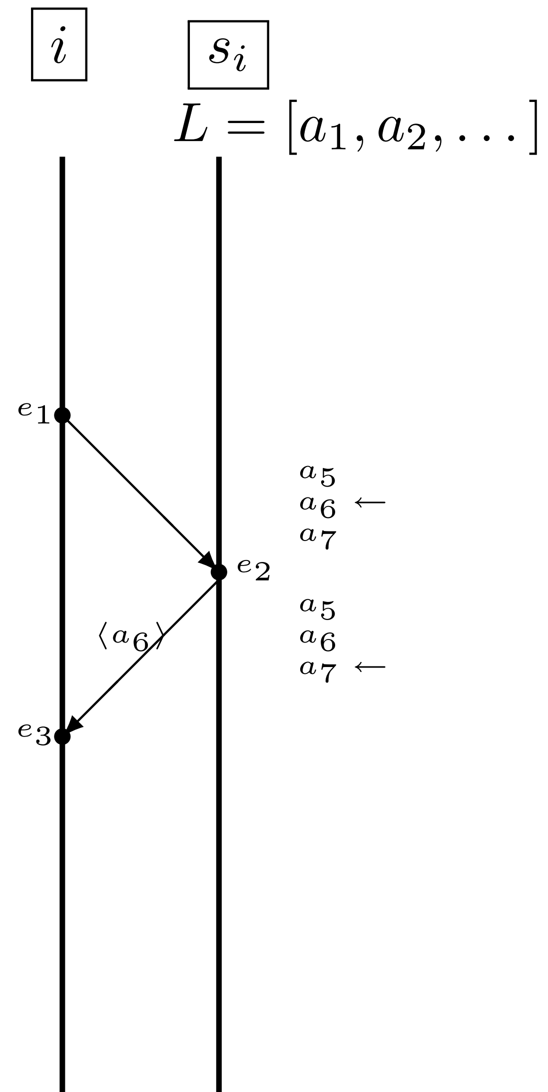

NonceServer(i,ats) $\equiv$ let k = rcv(l^i, $nonce$) in

at location s_i **initially** $ptr = 0$

at location s_i **initially** L = ats

effect of k **is** ptr:=ptr+1

k **sends on link** l_i :

if $ptr < len(L)$ then $\langle nonce, L[ptr] \rangle$ else **nil**

only [ ] **affects** L **at location** s_i

only $[k]$ **affects** ptr **at location** s_i

k **affects only** $[ptr]$ **at location** s_i

k **sends only on link/tags** $[\langle l_i, nonce \rangle]$

only $[k]$ **read** L **at location** s_i

19 Atoms & Independence

New Type *Atom*

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Permutation rule

From any judgement, $J(a, b, \ldots)$, in the logic, mentioning a finite set $a, b, \ldots$ of these names, we may infer the truth of $J(a', b', \ldots)$ whenever the mapping $a \mapsto a', b \mapsto b', \ldots$ is a permutation.

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E.g. Can't have $f(1) = tok(a)$

allowed: $f\{a, b\}(x) = \textbf{if } x = 1 \textbf{ then } tok(a) \textbf{ else } tok(b)$

New proposition $(x : T \| a)$, x *independent of* a (as a member of type T).

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We need only six rules to reason about independence.

Tactics automate most proofs.

Table 1: Rules for Independence(3 of 6)

independentTriviality

$$\frac{H \vdash x \in T \qquad H \vdash a \in Atom \qquad \text{closed x mentions no names}}{H \vdash (x : T \| a)}$$

independentBase

$$\frac{H \vdash \neg(x = a \in Atom)}{H \vdash (x : Atom \| a)}$$

independentApplication

$$\frac{H \vdash (f : (v : A \rightarrow B) \| a) \qquad H \vdash (x : A \| a)}{H \vdash (f(x) : B[x/v] \| a)}$$

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For all $e : E$ and $a : Atom$

$$(\mathbf{program}(loc(e)) \| a) \;\wedge\; (\forall e' <_{loc} e.\ isrcv(e') \;\Rightarrow\; (val(e') \| a)) \;\Rightarrow\; (\mathbf{state\ when}\ e \| a)$$

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Proof is by induction on $<$ and uses the rules about independence, especially "independentApplication"